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Nonlinear Classical Theory of Electromagnetism

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Abstract

A topological theory of electric charge is given. Einstein's criteria for the completion of classical electromagnetic theory are summarized and their relation to quantum theory and the principle of complementarity is indicated. The inhibiting effect that this principle has had on the development of physical thought is discussed. Developments in the theory of functions on nonlinear spaces provide the conceptual framework required for the completion of electromagnetism. The theory is based on an underlying field which is a continuous mapping of space-time into points on the two-sphere.

About 70 years ago Einstein characterized the elementary quantum of electricity e as a "stranger in Maxwell–Lorentz' electrodynamics" and, stressing the importance of Jean's observation that the combination e^2/hc forms a dimensionless quantity, he expressed the hope "that the same modification of the theory that will contain the elementary quantum e as a consequence, will also have as a consequence the quantum theory of radiation" (Einstein, 1909). He also criticized what he called "the disturbing dualism . . . which lies in the fact that the material point in Newton's sense and the field as continuum are used as elementary concepts side by side" (Schilpp, 1949). The required modification which would remove the fundamental distinction between electric charge and the electromagnetic field and at the same time have as a consequence the quantization of charge, necessarily involved nonlinear field equations. Since no method existed at that time by which this kind of field equation could be discovered, the problems of atomic structure and radiation were analyzed using the Maxwell–Lorentz conception of the electron as a material point coupled to a linear electromagnetic field. This analysis gave rise to the theory of quantum mechanics of atomic systems and later to quantum electrodynamics, which is the prototype for almost all subsequent theoretical developments in particle theory.

Quantum mechanics involves a more extreme form of duality than exists in classical electromagnetism. The mathematical framework of quantum mechanics

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requires that both a linear wave and a material point be associated with the *same* physical entity. The originators of the theory characterized this feature as an inherent limitation of language (Heisenberg, 1930). The incorporation of this idea as an epistemological principle into the professional training of physicists has had a profound effect on the subsequent development of the science. The student who has been educated to a certain point tries to form concepts of the electron and photon that can be successfully applied to the understanding of atomic structure and the interaction of matter and light. The language provided by the discipline he is studying is admittedly inadequate and unsatisfactory. Yet he is required to accept, as a matter of principle, the futility of any attempt to define in words and images the objects he studies. As a result, in order to continue his professional training, he must in effect agree not to expand the language of physics. Therefore, this principle, known as "complementarity" conveys an inherently negative view of language, one that implies the existence of essential truths that cannot be expressed in language. Thus there exists a similarly limited notion of mathematics' contribution to physics. The mathematician Robert Hermann identified the effect of this limitation when he characterized contemporary physicists as "so mathematically 'pragmatic' that they have largely lost the talent and taste for creative mathematical speculation that played such an important role in the past" (Hermann, 1977).

Since the formulation of quantum mechanics, progress in the theory of continuous mappings of intrinsically nonlinear spaces has provided a language and imagery inconceivable to the originators of "complementarity," whose notion of "wave" involved only linear spaces. (Although the equation of propagation might be nonlinear, the space of admissible initial conditions was always assumed to be linear.) Implicit in this development is the possibility that by framing a theory of elementary interactions utilizing this form of mathematics, one can free oneself of the limitations imposed by "complementarity." The rejection of "complementarity" does not challenge the validity of the uncertainty relations. This question can only be settled by careful analysis of measurement in terms of the new theory.

Motivated by these considerations and guided by the pioneering work of Finkelstein and Misner (1959) and Skyrme (1962), who first applied these mathematical results to particle physics, I have been successful in constructing a system of equations that imply Maxwell's equations. However, in this case, both the electromagnetic field and the charge-current density are derived from a single underlying field $\varphi(\bar{x}, t)$ which is a continuous mapping of points in space-time to points on the surface of the unit sphere, subject to the boundary condition $\varphi(\bar{x}, t) \to \varphi_0$, as $|\bar{x}| \to \infty$, where φ_0 is some fixed point on the sphere. Under these conditions the field admits homotopically invariant structures known as "kinks" (Finkelstein and Rubenstein, 1968).

If a, b are coordinates on the sphere, the underlying field can be represented locally by a pair of real-valued functions $a(\bar{x}, t), b(\bar{x}, t)$. The electromagnetic field is given by

$$F_{ij} = \frac{K(a,b)}{4\pi} \frac{\partial(a,b)}{\partial(x^i, x^j)} \tag{1}$$

where $K(a, b)$ is the area density on the sphere, $x^0 = t, x^1 = x, x^2 = y$, and $x^3 = z$. It can be shown that for any $\varphi(\bar{x}, t)$, the dual tensor

$$G^{ij} = \epsilon^{ijkl} F_{kl} \tag{2}$$

has vanishing divergence,

$$\frac{\partial G^{ij}}{\partial x^j} = 0 \tag{3}$$

Thus magnetic charges do not exist in this theory and the electromagnetic field can be derived from a vector potential A_i,

$$F_{ij} = \frac{\partial A_j}{\partial x^i} - \frac{\partial A_i}{\partial x^j} \tag{4}$$

The kink current density

$$J_i = g_{ij} G^{jk} A_k \tag{5}$$

where g_{ij} is the metric of flat space-time, is a conserved current:

$$\frac{\partial J_i}{\partial x_i} = \tfrac{1}{2} F_{ij} G^{ij} = 0 \tag{6}$$

This current has the remarkable property that the integral of J_0 over a region of space bounded by a surface on which $\varphi(\bar{x}, t)$ is a constant, is always an integer (Whitehead, 1947). Setting

$$\frac{\partial F_{ij}}{\partial x_i} = J_j \tag{7}$$

completes Maxwell's equations. This last step is the only equation that is not the result of a mathematical theorem. I point out that this equation is not invariant under the usual gauge transformation, since the current density depends explicitly on A_i. Also it is interesting to note that

$$\bar{A}(\bar{x}, t) = \boldsymbol{\epsilon} f(\bar{k} \cdot \bar{x} - kt), \qquad \boldsymbol{\epsilon} \cdot \bar{k} = 0, \qquad A_0 = 0 \tag{8}$$

is a solution for any function f. The current density J_i is identically zero in this case. However, a wave packet that is finite in the transverse directions will necessarily involve nonvanishing current density. Equation (6) implies that electric and magnetic fields are everywhere and always mutually perpendicular. Therefore, a singly charged stationary solution will have an intrinsic magnetic moment. Spatial inversion, $x_i \to g_{ik} x_k$, induces the transformations $A_i \to g_{ik} A_k$, $J_i \to -g_{ik} J_k$, and $F_{ij} \to g_{ik} g_{jm} F_{km}$. Time reversal, $x_i \to -g_{ik} x_k$, induces the transformations $A_i \to -g_{ik} A_k$, $J_i \to g_{ik} J_k$, and $F_{ij} \to g_{ik} g_{jm} F_{km}$.

The physical interpretation of these equations remains in a preliminary stage of development. The issues of immediate interest include the search for a suitable Lagrangian density, as well as expressions for energy-momentum density and angular momentum density. The dynamical content of (7) must be explored. Numerical methods have to be employed to find a stationary charged solution and solutions of finite extent that propagate like (8). However, it is already evident that the theory satisfies the criteria set by Einstein. The quantization of charge occurs naturally, and the notion of material point has been eliminated along with the essential distinction between the electromagnetic field and the electric current density, since both are derived from the same basic field $\varphi(\bar{x}, t)$. Thus, according to Einstein, we should also look for a quantum theory of radiation in these equations. Already the fundamental result usually associated with relativistic quantum field theory has been obtained: the existence of antiparticles. It is also worth pointing out that the kinks of this theory are capable of representing spin one-half fermions in the sense of Finkelstein and Rubenstein (1968).

These results, obtained by employing a nonlinear manifold as the range space of a classical field, imply a rejection of wave–particle duality. A single reality cannot be usefully or adequately defined by means of two disparate and separately inadequate concepts. The principle of complementarity is an illusion that was created to allow paradoxical interpretations of certain experimental facts to coexist in the preexisting conceptual framework. Rather than attempting a redefinition of the basic elements of their conceptual system, a majority of scientists preferred to avoid and thus obscure the problem inherent in wave–particle duality by attributing it to a supposed limitation of language. The ensuing prohibitions on scientific investigation present a severe impediment to the evolution of physical thought. In order to develop physical theory further one must disregard the restrictions imposed by the authoritarian and nihilistic philosophy underlying much of the current physics tradition.

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